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HCM-003-001543

Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

October - 2017

S - 502 : Statistics

(Mathematical Statistics) (New Course)

Faculty Code : 003

Subject Code : 001543

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Q.1 carries 20 marks.
(2) Q. 2 and 3 carries 25 marks each.
(3) Students can use their own scientific calculator.

1 Filling the blanks and short questions : (Each 1 mark) **20**

(1) If two independent variates $X_1 \sim N\left(\mu_1, \sigma_1^2\right)$ and $X_2 \sim N\left(\mu_2, \sigma_2^2\right)$ then $X_1 + X_2$ is dstributed as ________

(2) ________ is a moment generating function of Standard Normal distribution.

(3) ________ is a moment generating function of $\gamma(p)$.

(4) If x follows Gamma distribution with parameter p then $\mu_4 = k_4 + 3k_2^2$ is ________

(5) If two independent variates $X_1 \sim \gamma(n_1)$ and $X_2 \sim \gamma(n_2)$ then $\frac{X_1}{X_1 + X_2}$ is distributed as ________

(6) If $x_1, x_2, x_3, \ldots, x_n$ is independent log normal variates then its products also ________

(7) Measured of Kurtosis coefficient for Chi-square distribution are ________ and ________

(8) Weibull distribution has application in ________.

(9) If χ_1^2 and χ_2^2 are two independent Chi-square variates with d.f. n_1 and n_2 respectively, then the distribution of $\frac{\chi_1^2}{\chi_2^2}$ is ________

(10) If χ_1^2 and χ_2^2 are two independent Chi-square variates with d.f. n_1 and n_2 respectively, then the distribution of $\dfrac{\chi_1^2}{\chi_1^2+\chi_2^2}$ is __________

(11) t_n – distribution tends to normal if __________

(12) t – distribution with 1 d.f. reduces to __________

(13) If $X \sim N(0,1)$ and $Y \sim \chi_n^2$, the statistic $\dfrac{\sqrt{n}X}{\sqrt{Y}}$ is distributed as __________.

(14) The marks X and Y secured by examinees in Statistics and Mathematics will follow __________ distribution.

(15) Given a joint Bivariate Normal distribution of X, Y as $\text{BVN}\left(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho\right)$, the marginal distribution $f_x(X) =$ __________

(16) A measure of linear association of a varible say, X_1 with a number of other variables $X_2, X_3, X_4, \ldots X_k$ is known as __________

(17) The range of multiple correlation coefficient R is __________

(18) If X_1, X_2 and X_3 are three variables, the regression planes X_1 on X_2, X_3; X_2 on X_1, X_3 and X_3 on X_1, X_3 are coincident iff $r_{12}^2 + r_{13}^2 + r_{23}^2 - 2r_{12}\, r_{13}\, r_{23} =$ __________

(19) Partial correlation coefficients is a measure of association between two variables __________ the common effect of the rest of the variable.

(20) __________ is a characteristic function of Binomial distribution.

2 (A) Write the answer any **three** : (Each 2 marks) **6**

(1) Prove that $\sigma_{3.12}^2 = \dfrac{\sigma_3^2\left(1 - r_{12}^2 - r_{23}^2 - r_{13}^2 + 2r_{12}\, r_{23}\, r_{13}\right)}{\left(1 - r_{12}^2\right)}$

(2) If $u = \dfrac{x-a}{h}$, a and h being constants then $\phi_u(t) = e^{(-iat/h)}\, \phi_x(t/h)$

(3) Define truncated distribution.

(4) Usual notion of multiple correlation and multiple regression, prove that $\sum X_{1.23} x_2 = 0$

(5) Prove that $b_{12.3} = \dfrac{b_{12} - b_{13} b_{23}}{1 - b_{13} b_{23}}$

(6) In trivariate distribution it is found that $r_{12} = 0.59$, $r_{13} = 0.46$ and $r_{23} = 0.77$. Find (i) $r_{12.3}$ (ii) $R_{1.23}$

(B) Write the answer any **three** : (Each 3 marks) **9**

(1) Prove that $\mu_r = (-i)^r \left[\dfrac{d^r}{dt^r} \phi_u(t) \right]_{t=0}$; where $u = x - \mu$

(2) Obtain MGF of Normal distribution.

(3) Define Exponential distribution and obtain its MGF. From MGF obtain its mean and variance.

(4) Define truncated Poisson distribution and also obtain its mean and variance.

(5) Usual notation of multiple correlation and multiple regression, prove that $b_{12} = \dfrac{b_{12.3} + b_{13.2}\, b_{32.1}}{1 - b_{13.2}\, b_{31.2}}$

(6) Usual notation of multiple correlation and multiple regression, prove that $\sigma_{1.23}^2 = \sigma_1^2 \left(1 - r_{12}^2\right)\left(1 - r_{13.2}^2\right)$

(C) Write the answer any **two** : (Each 5 marks) **10**

(1) Obtain conditional distribution of y when x is given for Bi-variate distribution.

(2) Derive t-distribution.

(3) Derive χ^2 distribution and show that $3\beta_2 - 2\beta_1 + 6 = 0$.

(4) Obtain marginal distribution of x for Bi-variate distribution.

(5) Usual notation of multiple correlation and multiple regression, prove that $R_{1.23}^2 = \dfrac{r_{12}^2 + r_{13}^2 - 2 r_{12}\, r_{23}\, r_{13}}{1 - r_{23}^2}$

3 (A) Write the answer any **three** : (Each 2 marks) **6**

(1) Define Beta-I and Beta-II distribution.

(2) Obtain characteristic function of Poisson distribution with parameter λ.

(3) Define Weibul distribution.

(4) Usual notion of multiple correlation and multiple regression, prove that $\sum X_{1.2} X_{3.12} = 0$

(5) Define Convergence in Probability.

(6) In trivariate distribution it is found that $\sigma_1 = \sigma_3 = 2.7, \sigma_2 = 2.4, r_{12} = 0.28, r_{23} = 0.49, r_{31} = 0.51$ find (i) $b_{32.1}$ (ii) $\sigma_{2.31}$

(B) Write the answer any **three** : (Each 3 marks) **9**

(1) Prove that $\mu_r^{'} = (-i)^r \left[\frac{d^r}{dt^r} \phi_x(t) \right]_{t=0}$

(2) Obtain Probability density function for the characteristic function $\phi_x(t) = p\left(1 - qe^{it}\right)^{-1}$

(3) Obtain mean and variance of Uniform Distribution.

(4) Define truncated Binomial distribution and also obtain its mean and variance.

(5) Usual notation of multiple correlation and multiple regression, prove that if $r_{12} = r_{23} = r_{31} = r$ then $R_{1.23} = R_{2.31} = R_{3.12} = \frac{\sqrt{2}\, r}{\sqrt{1+r}}$

(6) Usual notation of multiple correlation and multiple regression, prove that $b_{12.3} b_{23.1} b_{31.2} = r_{12.3} r_{23.1} r_{31.2}$

(C) Write the answer any **two** : (Each 5 marks) **10**

(1) Obtain MGF of Gamma distribution with parameters α and p. Also show that $3\beta_1 - 2\beta_2 + 6 = 0$.

(2) Derive Normal distribution.

(3) Derive F-distribution.

(4) State and Prove that Chebichev's inequality.

(5) Usual notation of multiple correlation and multiple regression, prove that $r_{12.3} = \frac{r_{12} - r_{13} r_{23}}{\sqrt{\left(1 - r_{13}^2\right)\left(1 - r_{23}^2\right)}}$
